

# Axion and Right-handed Neutrino in the Minimal SUSY SO(10) Model

---

**Takeshi Fukuyama, Tatsuru Kikuchi**

*Department of Physics, Ritsumeikan University*

*Kusatsu, Shiga, 525-8577 Japan*

*E-mail: fukuyama@se.ritsumei.ac.jp, rp009979@se.ritsumei.ac.jp*

**ABSTRACT:** The connection between the axion and right-handed neutrinos is explored in the framework of the minimal SUSY SO(10) model. The former is related to the Peccei-Quinn (PQ) solution to the strong CP problem and the latter is to the light Majorana neutrinos through the see-saw mechanism. In this model, a relative phase between  $(\mathbf{10}, \mathbf{1}, \mathbf{3})$  ( $\equiv \bar{\Delta}_R$ )  $\subset \overline{\mathbf{126}}$  and  $(\overline{\mathbf{10}}, \mathbf{1}, \mathbf{3})$  ( $\equiv \Delta_R$ )  $\subset \mathbf{126}$  multiplets of  $SU(4) \times SU(2)_L \times SU(2)_R \subset SO(10)$  becomes a physical degree of freedom identified with the axion. Then, the PQ symmetry breaking scale ( $\Lambda_{PQ}$ ) and the  $B-L$  symmetry breaking scale ( $\Lambda_{B-L}$ ) coincide through the VEV of  $\bar{\Delta}_R$ . The scalar partner of the lightest right-handed neutrino is regarded as the inflaton, which gives a consistent density fluctuation for the CMB.

**KEYWORDS:** Cosmology of Theories beyond the SM, Beyond Standard Model, GUT, Neutrino Physics.

---

## Contents

<b>1. Introduction</b>	<b>1</b>
<b>2. SO(10) model</b>	<b>3</b>
<b>3. Conclusion</b>	<b>5</b>

---

### 1. Introduction

The supersymmetric (SUSY) grand unified theory (GUT) has received particular attention over the last decade. In particular, with the particle content of the minimal supersymmetric standard model (MSSM), the three gauge coupling constants converge at the GUT scale  $M_{\text{GUT}} \simeq 2 \times 10^{16}$  [GeV]. In addition to this, recent progress in neutrino physics makes SO(10) a plausible candidate for GUTs, since it naturally incorporates the see-saw mechanism [1] that can naturally explain the lightness of the neutrinos. In particular, minimal SO(10) models are very natural to realize since it not only reproduces the low energy experimental data but also predicts the unobserved values of absolute masses of light neutrinos and heavy right-handed neutrinos and the full MNS mixing matrix very restrictively [2, 3, 4, 5, 6, 7]. Recently well-confirmed atmospheric and solar neutrino oscillation data with the see-saw mechanism indicates the scale of the right-handed neutrinos. The typical prediction in the minimal SO(10) models suggests the scale to be [4]

$$M_{R1} \simeq 1.2 \times 10^{11} \text{ [GeV]}, \quad M_{R2} \simeq 1.8 \times 10^{12} \text{ [GeV]}, \quad M_{R3} \simeq 8.3 \times 10^{12} \text{ [GeV]}. \quad (1.1)$$

Even, after this analysis had been performed, there has been remarkable progress in the solar neutrino oscillation data from the KamLAND experiment [9], which does not affect so seriously the above values. On the other hand, one of the most likely solutions to the strong CP problem, the Peccei-Quinn (PQ) solution, gives us more interesting information at such an intermediate scale. Namely, in the PQ solution, we have probably an invisible axion with a decay constant  $f_a$  that is severely constrained from astrophysics as:

$$10^{9 \pm 1} \text{ [GeV]} \lesssim f_a \lesssim 10^{12 \pm 1} \text{ [GeV]}. \quad (1.2)$$

This range is very similar to the scale of the right-handed neutrinos. Hence it seems possible that there is some deep connection between the two physical scales, the PQ symmetry breaking scale ( $\Lambda_{\text{PQ}}$ ) and the  $B - L$  symmetry breaking scale ( $\Lambda_{B-L}$ ). In this paper, we explore the connections of the axion physics and the right-handed neutrino physics with the help of the minimal SO(10) grand unification model. In SO(10) models, the gauged  $B - L$  symmetry can play the role of protecting the right-handed Majorana neutrino masses from

becoming as large as the GUT scale. In this sense, the SO(10) symmetry is a necessary gauge symmetry for the GUT to argue about the nature of the intermediate energy scale about  $10^{13}$  [GeV].

In the following, we shall only illustrate the essence of the mechanism to connect the axion and right-handed neutrinos. First, the right-handed neutrino masses are generated through the following type of Yukawa interaction,

$$W = Y_{126}^{ij} \bar{\Delta}_R \nu_{Ri}^T C^{-1} \nu_{Rj} . \quad (1.3)$$

This gives the Majorana masses for the right-handed neutrinos,  $M_R^{ij} = Y_{126}^{ij} \langle \bar{\Delta}_R \rangle$ . In general, we can assign a global  $U(1)_{PQ}$  charge to these fields. For instance,  $PQ[\bar{\Delta}_R] = +2$ ,  $PQ[\nu_R] = -1$ . Then after giving rise to the VEV of  $\bar{\Delta}_R$ , the global  $U(1)_{PQ}$  symmetry would be spontaneously broken and there appears a pseudo-NG boson that is later understood as the axion. The scalar potential of the  $\bar{\Delta}_R$  field includes the mixing term with the electroweak Higgs doublets,

$$V = \lambda \bar{\Delta}_R \Delta_R H_{126} H_{10} . \quad (1.4)$$

Here  $H_{10} \equiv (\mathbf{1}, \mathbf{2}, \mathbf{2})$  and  $H_{126} \equiv (\mathbf{15}, \mathbf{2}, \mathbf{2})$  are the  $SU(2)_L$  bi-doublet Higgs fields arising from the **10** and **126** multiplets of SO(10), respectively, and  $\Delta_R$  is required for the anomaly cancellation. A linear combination of  $H_{10}$  and  $H_{126}$  Higgs fields becomes the MSSM Higgs doublets  $H_u$  and  $H_d$  that cause the correct electroweak symmetry breaking,  $SU(2)_L \times U(1)_Y \rightarrow U(1)_{\text{em}}$ . This potential would cause a connection between intermediate scale physics and the electroweak scale. Since the fields  $H_u$  and  $H_d$ , or equivalently  $H_{10}$  and  $H_{126}$  have couplings to the quarks and leptons, a rotation of the Higgs fields  $H_{10} \rightarrow H_{10} \exp(+2i\theta)$  and  $H_{126} \rightarrow H_{126} \exp(+2i\theta)$  gives a chiral rotation of the quarks and leptons  $\{q_L, u_R^c, d_R^c, \ell_L, \nu_R^c, e_R^c\} \rightarrow \exp(-i\theta)\{q_L, u_R^c, d_R^c, \ell_L, \nu_R^c, e_R^c\}$ . Such a non-trivial transformation indicates an anomalous symmetry and it induces an anomalous coupling of the pseudo-NG boson  $a(x)$  to the gluon field.

$$\mathcal{L} = \frac{a(x)}{f_a} \frac{g_s^2}{32\pi^2} G_{\mu\nu}^A \tilde{G}_A^{\mu\nu} , \quad (1.5)$$

where  $g_s$  is the  $SU(3)_c$  gauge coupling constant,  $G_{\mu\nu}^A$  is the gluon field strength and  $\tilde{G}_A^{\mu\nu} \equiv \frac{1}{2} \epsilon^{\mu\nu\rho\sigma} G_{\rho\sigma}^A$ . This kind of interaction is used to solve the strong CP problem [10]. Then the interaction of the axion with the quarks and leptons is given by

$$\mathcal{L} = \frac{a(x)}{f_a} \partial_\mu J^\mu , \quad (1.6)$$

where  $J^\mu$  is a conserved current associated with the global  $U(1)_{PQ}$  symmetry

$$J^\mu = f_a \partial^\mu a(x) + 2 \sin^2 \beta \bar{u}_i \gamma^\mu \gamma_5 u_i + 2 \cos^2 \beta \bar{d}_i \gamma^\mu \gamma_5 d_i + 2 \cos^2 \beta \bar{e}_i \gamma^\mu \gamma_5 e_i . \quad (1.7)$$

with  $\tan \beta \equiv \langle H_u \rangle / \langle H_d \rangle$ . As is usual for the pseudo-NG bosons, the mass of the axion is inversely proportional to the decay constant  $f_a$  as

$$m_a = 0.62 \times 10^{-6} [\text{eV}] \times \frac{10^{13} [\text{GeV}]}{f_a} . \quad (1.8)$$

Thus, the right-handed neutrino mass scale suggested from the recent neutrino oscillation data implies the appearance of an invisible axion with mass

$$m_a \simeq 7.5 \times 10^{-5} \text{ [eV]} . \quad (1.9)$$

Though there have already been many models of the axion [11, 12, 13], and their applications to GUT models [14, 15, 16, 17]. In this paper, we consider now to incorporate the axion into the minimal  $\text{SO}(10)$  model.

## 2. $\text{SO}(10)$ model

In order to realize the axion in the minimal  $\text{SO}(10)$  model, let us denote the right-handed neutrino superfield as  $N \equiv \nu_R$ . As mentioned in the introduction, the masses of the right-handed neutrinos are given by an  $SU(2)_R$  triplet Higgs field  $\bar{\Delta}_R$ . In the minimal  $\text{SO}(10)$  model [2, 3, 4, 5, 6, 7], such a triplet field  $\bar{\Delta}_R$  can naturally be obtained from the  $\bar{\Delta} = \bar{\mathbf{126}}$  Higgs field. In order to avoid a heavy axion mass, which does not solve the strong CP problem, we also impose a discrete symmetry  $\mathbb{Z}_3$ . The corresponding charges with regards to this  $\mathbb{Z}_3$  symmetry are listed in Table 1. Then, the  $\text{SO}(10) \times \mathbb{Z}_3$  invariant superpotential is given by

$$W = \Psi_i (Y_{10}^{ij} H + Y_{126}^{ij} \bar{\Delta}) \Psi_j + m_1 \bar{\Delta} \Delta + m_2 \Phi^2 + \lambda_1 \bar{\Delta} \Delta \Phi + \lambda_2 \Delta H \Phi + \lambda_3 \Phi^3 , \quad (2.1)$$

where  $\Psi_i$  is a **16**-dimensional matter multiplet,  $H$  is a **10**-dimensional multiplet which essentially gives a large top Yukawa coupling and  $\Phi$  is a **210**-dimensional multiplet that is used to break the  $\text{SO}(10)$  gauge symmetry. The details of this potential can be found in [7, 8].

The essential point in this framework to generate the PQ axion is as follows: the **126** and the **126** are independent fields required in order to preserve SUSY, but they always appear in pairs, and the SUSY vacuum condition (D-flat condition) can never determine the relative phase degree of freedom:

$$|\langle \bar{\Delta}_R \rangle|^2 - |\langle \Delta_R \rangle|^2 = 0 . \quad (2.2)$$

This means, the relative phase remains as a physical degree of freedom, the so called pseudo-NG boson. Schematically, we can write this fact as follows:

$$\langle \bar{\Delta}_R \rangle \sim \langle \Delta_R \rangle \times \exp(i\Theta) , \quad (2.3)$$

where the argument field or the pseudo-NG boson  $\Theta$  can be regarded as the axion. It gives a connection between the  $U(1)_{B-L}$  symmetry breaking scale ( $\Lambda_{B-L}$ ) and the  $U(1)_{\text{PQ}}$  symmetry breaking scale ( $\Lambda_{\text{PQ}}$ )<sup>1</sup>. That is one of our main conclusions in this article.

---

<sup>1</sup>Note that since these two fields  $\bar{\Delta}_R$  and  $\Delta_R$  are completely independent, one of which is used to break the  $B - L$  symmetry and the other can be used to break the PQ symmetry as well. Remarkably, the former symmetry is gauged in  $\text{SO}(10)$  although the latter one is ungauged, hence one of the NG bosons residing in the above fields is absorbed into the  $B - L$  gauge boson, but the other remains as a physical degree of freedom, the axion.

fields	PQ charges	$\mathbb{Z}_3$ charges
$\Psi$	-1	$\omega^2$
$H$	+2	$\omega^2$
$\bar{\Delta}$	+2	$\omega^2$
$\Delta$	-2	$\omega$
$\Phi$	0	1

**Table 1:** PQ and  $\mathbb{Z}_3$  charges of the fields ( $\omega^3 = 1$ ).

After the SO(10) symmetry breaking, we have the following superpotential for the matter multiplets:

$$\begin{aligned}
W = & u_{Ri}^c \left( Y_{10}^{ij} H_{10}^u + Y_{126}^{ij} H_{126}^u \right) q_{Lj} + d_{Ri}^c \left( Y_{10}^{ij} H_{10}^d + Y_{126}^{ij} H_{126}^d \right) q_{Lj} \\
& + N_i^c \left( Y_{10}^{ij} H_{10}^u - 3 Y_{126}^{ij} H_{126}^u \right) \ell_{Lj} + e_{Ri}^c \left( Y_{10}^{ij} H_{10}^d - 3 Y_{126}^{ij} H_{126}^d \right) \ell_{Lj} \\
& + Y_{126}^{ij} N_i^c N_j^c \bar{\Delta}_R .
\end{aligned} \tag{2.4}$$

Each field can have the PQ charges as listed in Table 1. In addition to this, we have the soft SUSY breaking terms defined as follows:

$$V_{\text{SOFT}} = m_{\tilde{N}_i}^2 |\tilde{N}_i|^2 + m_{\bar{\Delta}}^2 |\bar{\Delta}_R|^2 + m_{\Delta}^2 |\Delta_R|^2 + \left( A_N^{ij} \bar{\Delta}_R \tilde{N}_i \tilde{N}_j + \text{h.c.} \right) , \tag{2.5}$$

where  $A_N^{ij}$  is the tri-linear coupling constant which is assumed to be proportional to the Yukawa coupling constant  $Y_{126}^{ij}$ ,  $A_N^{ij} = m_{3/2} Y_{126}^{ij}$ . From the superpotential given above, we can calculate the scalar potential in the usual way:

$$V = \left| \frac{\partial W}{\partial \bar{\Delta}_R} \right|^2 + \left| \frac{\partial W}{\partial \tilde{N}} \right|^2 + V_{\text{SOFT}} , \tag{2.6}$$

that is,

$$V = m_{\tilde{N}_i}^2 |\tilde{N}_i|^2 + (M_{\text{GUT}}^2 + m_{\bar{\Delta}}^2) |\bar{\Delta}_R|^2 + \left\{ \left( Y_{126}^{ij} M_{\text{GUT}} + A_N^{ij} \right) \bar{\Delta}_R \tilde{N}_i^* \tilde{N}_j + \text{h.c.} \right\} + \dots . \tag{2.7}$$

We regard the scalar partner of the lightest right-handed neutrino (sneutrino)  $\tilde{N}_1$  as the inflaton, that is, we consider the sneutrino inflation scenario [18]. In this case, a condensation of the scalar field  $\langle \tilde{N}_1 \rangle$  causes the inflation and the successive reheating processes.

Then the above potential drives the sneutrino  $\tilde{N}_1$  (a hybrid inflation [19]) and it determines the inflaton (sneutrino) mass to be around  $m_{\text{inf}} \simeq (M_{\text{GUT}} M_{R1})^{1/2} \simeq 5.7 \times 10^{13}$  [GeV]. The mass scale of the sneutrino as the inflaton is the appropriate one for the time of coherent oscillation until the end of inflation  $H \simeq \Gamma_{\tilde{N}_1}$  ( $H$ : Hubble parameter), namely, it leads to the COBE normalization of the primordial density fluctuation [20]

$$\frac{\delta T}{T} \simeq \left( \frac{m_{\text{inf}}}{M_P} \right) \simeq 10^{-5} . \tag{2.8}$$

Here the tree level sneutrino decay rate is given by

$$\Gamma_{\tilde{N}_1} \simeq \frac{1}{4\pi} \left( Y_\nu Y_\nu^\dagger \right)^{11} M_{R1} \simeq 6.1 \times 10^7 \text{ [GeV]}, \quad (2.9)$$

where  $Y_\nu$  is the neutrino Dirac Yukawa coupling matrix, and we took the typical value of  $\left( Y_\nu Y_\nu^\dagger \right)^{11} \simeq 4.7 \times 10^{-3}$ . Thus the reheating temperature in this model is given by [21]

$$T_R = \left( \frac{45 M_P^2}{2\pi^2 g_*} \right)^{1/4} \left( \Gamma_{\tilde{N}_1} \right)^{1/2} \simeq 4.0 \times 10^{12} \text{ [GeV]}. \quad (2.10)$$

After giving rise to the PQ symmetry breaking VEV of the Higgs,

$$\langle \bar{\Delta}_R \rangle \simeq 8.3 \times 10^{12} \text{ [GeV]}, \quad (2.11)$$

the argument of  $\bar{\Delta}_R$  can be regarded as the PQ field or an invisible axion,  $a(x) \equiv f_a \times [\arg(\bar{\Delta}_R) - \arg(\Delta_R)]$  with the decay constant  $f_a = |\langle \bar{\Delta}_R \rangle| \simeq 8.3 \times 10^{12} \text{ [GeV]}$ .

Finally, it should be noted that the gauged  $B - L$  symmetry included in the SO(10) symmetry protects the sneutrinos from having large initial values along the existing  $B - L$  flat direction. Therefore we can not incorporate the simple chaotic inflation scenario [24] into the SO(10) models, and we must use the hybrid inflation model. Recent WMAP data also supports the fact that multi-field hybrid inflation models are preferable to the single field chaotic inflation model [25].

### 3. Conclusion

It has been found that the Peccei-Quinn solution to the strong CP or the axion problem can be embedded in the same framework of the right-handed neutrino sector. A complete correspondence between the axion and the Higgs that gives a mass for the right-handed neutrino has been obtained based on an SO(10) model. The relative phase between  $\bar{\Delta}_R$  and  $\Delta_R$  can be identified with the axion itself. The resemblances among the symmetry breaking scale of the PQ symmetry and the  $B - L$  symmetry or the right-handed neutrino mass scale are thus well founded. An axion mass consistent with the neutrino oscillation data is found to be  $m_a \simeq 7.5 \times 10^{-5} \text{ [eV]}$ . In addition, assuming the sneutrino as the inflaton can naturally be embedded into the model with a sneutrino mass around  $\simeq 10^{13} \text{ [GeV]}$ , which is consistent with the density fluctuation of the CMB.

### Acknowledgments

We are grateful to W. Naylor for useful comments. The work of T.F and T.K is supported in part by the Grant-in-Aid for Scientific Research from the Ministry of Education, Science and Culture of Japan (#16540269). The work of T.K was supported by the Research Fellowship of the Japan Society for the Promotion of Science (# 7336).

## References

- [1] T. Yanagida, in Proceedings of the workshop on the Unified Theory and Baryon Number in the Universe, edited by O. Sawada and A. Sugamoto (KEK, Tsukuba, 1979); M. Gell-Mann, P. Ramond, and R. Slansky, in Supergravity, edited by D. Freedman and P. van Nieuwenhuizen (North-Holland, Amsterdam 1979); R.N. Mohapatra and G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980).
- [2] K.S. Babu and R.N. Mohapatra, Phys. Rev. Lett. **70**, 2845 (1993).
- [3] K. Matsuda, Y. Koide and T. Fukuyama, Phys. Rev. **D 64**, 053015 (2001) [arXiv:hep-ph/0010026]; K. Matsuda, Y. Koide, T. Fukuyama and H. Nishiura, Phys. Rev. **D 65**, 033008 (2002) [arXiv:hep-ph/0108202].
- [4] T. Fukuyama and N. Okada, JHEP **0211**, 011 (2002) [arXiv:hep-ph/0205066].
- [5] B. Bajc, G. Senjanović and F. Vissani, Phys. Rev. Lett. **90**, 051802 (2003) [arXiv:hep-ph/0210207]; H.S. Goh, R.N. Mohapatra and S.P. Ng, Phys. Lett. **B 570**, 215 (2003) [arXiv:hep-ph/0303055]; B. Dutta, Y. Mimura and R.N. Mohapatra, Phys. Rev. **D 69**, 115014 (2004) [arXiv:hep-ph/0402113].
- [6] C.S. Aulakh, B. Bajc, A. Melfo, G. Senjanović and F. Vissani, Phys. Lett. **B 588**, 196 (2004) [arXiv:hep-ph/0306242].
- [7] T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, [arXiv:hep-ph/0401213].
- [8] T. Fukuyama, A. Ilakovac, T. Kikuchi, S. Meljanac and N. Okada, J. Math. Phys. **46**, 033505 (2005) [arXiv:hep-ph/0405300].
- [9] K. Eguchi *et al.* [KamLAND Collaboration], Phys. Rev. Lett. **90**, 021802 (2003) [arXiv:hep-ex/0212021].
- [10] R.D. Peccei, H.R. Quinn, Phys. Rev. Lett. **38**, 1440 (1977); Phys. Rev. **D 16**, 1791 (1977).
- [11] S. Weinberg, Phys. Rev. Lett. **40**, 223 (1978); F. Wilczek, Phys. Rev. Lett. **40**, 279 (1978).
- [12] J. E. Kim, Phys. Rev. Lett. **43**, 103 (1979); M. Shifman, A. Vainstein and V. Zakharov, Nucl. Phys. **B 166**, 493 (1980).
- [13] A. Zhitnitsky, Sov. J. Nucl. Phys. **31**, 60 (1980); M. Dine and W. Fishler, and M. Srednicki, Phys. Lett. **B 104**, 199 (1981).
- [14] M. B. Wise, H. Georgi and S. L. Glashow, Phys. Rev. Lett. **47** (1981) 402.
- [15] H. P. Nilles and S. Raby, Nucl. Phys. B **198** (1982) 102.
- [16] R. N. Mohapatra and G. Senjanović, Z. Phys. C **17** (1983) 53.
- [17] L. J. Hall and S. Raby, Phys. Rev. D **51** (1995) 6524 [arXiv:hep-ph/9501298].
- [18] H. Murayama, H. Suzuki, T. Yanagida, J. Yokoyama, Phys. Rev. Lett. **70**, 1912, (1993); J.R. Ellis, M. Raidal, T. Yanagida, Phys. Lett. **B 581** 9, (2004).
- [19] A. Linde, Phys. Rev. **D 49**, 748 (1994).
- [20] D.S. Salopek, Phys. Rev. Lett. **69**, 3602 (1992).
- [21] This value of the reheating temperature is a little bit high in considering the gravitino problem with the hadronic decay [22]. To avoid such a difficulty, we just assume the heavy gravitino  $m_{3/2} \gtrsim 100$  [TeV] motivated by the anomaly mediated SUSY breaking (AMSB) scenario [23].

- [22] M. Kawasaki, K. Kohri and T. Moroi, [arXiv:astro-ph/0402490]; [arXiv:astro-ph/0408426].
- [23] L. Randall and R. Sundrum, Nucl. Phys. **B 557**, 79 (1999); G.F. Giudice, M.A. Luty, H. Murayama and R. Rattazzi, JHEP **9812**, 027 (1998).
- [24] A. Linde, Phys. Lett. **B 129**, 177 (1983).
- [25] V. Barger, H.-S. Lee, D. Marfatia, Phys. Lett. **B 565**, 33 (2003); W.H. Kinney, E.W. Kolb, A. Melchiorri, A. Riotto, Phys. Rev. **D 69**, 103516 (2004).